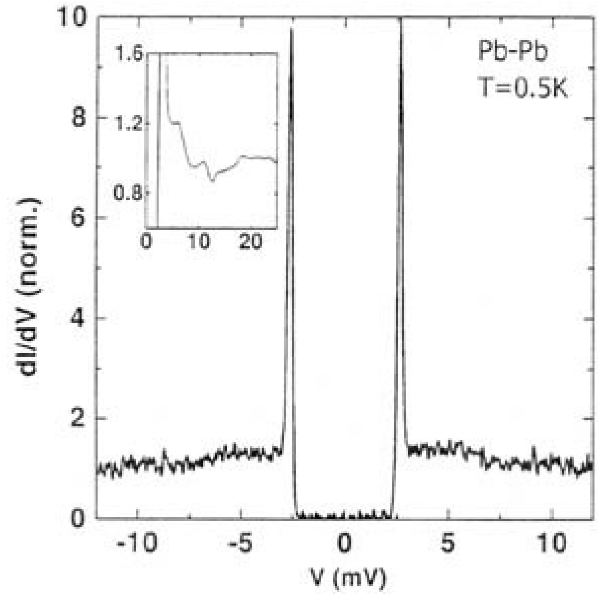

FIG. 1: Tunneling DOS of lead. Taken from H. Suderow, et al., Physica C, vol. 369, pp. 106 (2002).

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Problem set - 1

due: Friday Oct 11th by 5pm.

1. Electronic heat capacity of a metal.

   Approximate the density of states of a metal, $\rho(\epsilon_{\vec{k}}) = \rho_0$, as constant for $\epsilon(\epsilon_{\vec{k}} \geq 0)$, with no negative energy states.

   (a) Write down the integral that describes the energy of an electron gas with Fermi energy E_F, at a low temperature $T \ll E_F$, and filling a volume V.

   (b) What is the heat capacity of the electron gas to leading order in temperature?

   For the above you could use geometric arguments, or the Sommerfeld expansion which is described in a note posted on the website.

2. Electronic heat capacity of a superconductor. The low-T energy spectrum of a metal which turns superconducting is unique. A gap opens up about the fermi surface, and the excitations of the superconductor are bogolubov quasiparticles, that could appear in any of the electronic original k-states. The excitation energies of the Bogolubov quasiparticles (BQP's) are:

$$E_{\vec{k}} = \sqrt{\Delta^2 + (\epsilon_{\vec{k}} - E_F)^2} \tag{1}$$

   Each k-state supports two degenerate Bogolubov quasiparticles due to spin. $\epsilon_{\vec{k}}$ is the kinetic energy of unpaired electrons at momentum $\vec{k}$.

   Within the semi-conductor model of superconductors, we think of the BQP's with $\epsilon_{\vec{k}} > E_F$ as electronic excitations - not so different then juct excited electrons. The BQP's with $\epsilon_{\vec{k}} < E_F$ we think of as hole excitations in an otherwise filled Fermi sea. This implies thinking of the superconductor as if it is a semiconductor for the electronic energies being:

$$\tilde{\epsilon}_{\vec{k}} = \begin{cases} E_F + \sqrt{\Delta^2 + (\epsilon_{\vec{k}} - E_F)^2} & \epsilon_{\vec{k}} > E_F \\ E_F - \sqrt{\Delta^2 + (\epsilon_{\vec{k}} - E_F)^2} & \epsilon_{\vec{k}} < E_F \end{cases} \tag{2}$$

   (a) Show that the effective electronic energy for the semiconductor model, Eq. (2), reduces to the bare electronic kinetic energy when $\Delta \to 0$.

(b) Approximate the metallic density of states (in the abscence of superconductivity), $\rho(\epsilon_{\vec{k}}) = \rho_0$, as constant. Show that the DOS of the superconductor is given by:

$$\rho(E) = \begin{cases} 0 & |E| < \Delta \\ \rho_0 \frac{|E|}{\sqrt{E^2 - \Delta^2}} & |E| \geq \Delta \end{cases} . \tag{3}$$

(c) (ungraded) The density of states is directly measurable using tunneling experiments. In these experiments the differenetial conductance, $\frac{dI}{dV}$ is measured as a function of tunnleing voltage V. The conductivity is linearly proportinal to the DOS at energy eV. Plot the density of states you obtained above, and compare it to Fig. 1.

(d) Use the FD distribution to find an expression for the energy stored in the electronic states assuming $T \ll \Delta$. You can leave the answer in integral form. Assume that ρ_0 describes the bear density of states of the electrons (in the absence of superconductivity) as long as $\epsilon_{\vec{k}} > 0$.

(e) What is the electronic heat capacity of the superconductor? Find the leading behavior of the heat capacity as a function of temperature upto numerical factors. Hint: check that only the regions of the spectrum closest to the fermi-energy contribute.

3. (Optional review problem) The degeneracy pressure of a Fermi gas. Consider a 3d box of volume V with free electrons of density n. What is the pressure the electrons exert on the edges of the box at $T = 0$?

   Recall that the grand canonical potential is given by: $-pV = -T \ln \mathcal{Z}$ where $\mathcal{Z} = \prod_{\vec{k},\sigma} \mathcal{Z}_k$ is the grand canonical ensemble partition function.