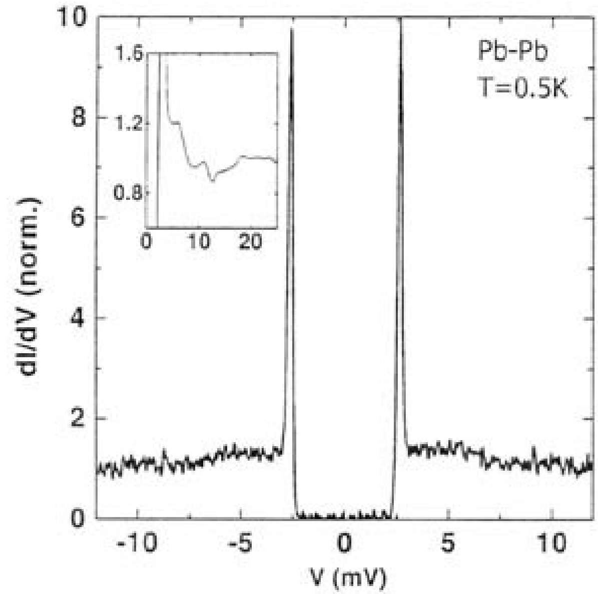

FIG. 1: Tunneling DOS of lead. Taken from H. Suderow, et al., Physica C, vol. 369, pp. 106 (2002).

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Problem set - 1

due: Friday Oct 12th by 5pm.

1. Electronic heat capacity of a metal

   Approximate the density of states of a metal, $\rho(\epsilon_{\vec{k}}) = \rho_0$, as constant for $\epsilon(\epsilon_{\vec{k}} \geq 0)$, with no negative energy states.

   (a) Write down the integral that describes the energy of an electron gas with Fermi energy E_F and at a low temperature $T \ll E_F$, and filling a volume V.

   (b) What is the heat capacity of the electron gas to leading order in temperature?

   For the above you could use geometric arguments, or the Sommerfeld expansion which is describe in a note posted on the website.

2. Electronic heat capacity of a superconductor. The low-T energy spectrum of a metal which turns superconducting is unique. A gap opens up about the fermi surface, and the excitation energies allowed are:

$$E = \pm\sqrt{\Delta^2 + (\epsilon_{\vec{k}} - E_F)^2} \tag{1}$$

   The sign of the excitation corresponds, roughly, to the sign of $(\epsilon_{\vec{k}} - E_F)$, and each state is doubly degenerate due to spin. $\epsilon_{\vec{k}}$ should be thought of as the kinetic energy of the electronic state $\vec{k}$ in the abscence of superconductivity.

   (a) Approximate the metallic density of states (in the abscence of superconductivity), $\rho(\epsilon_{\vec{k}}) = \rho_0$, as constant. Show that the DOS of the superconductor is given by:

$$\rho(E) = \begin{cases} 0 & |E| < \Delta \\ \rho_0 \frac{E}{\sqrt{E^2 - \Delta^2}} & |E| \geq \Delta \end{cases} . \tag{2}$$

   (b) (ungraded) The density of states is directly measurable using tunneling experiments. In these experiments the differenetial conductance, $\frac{dI}{dV}$ is measured as a function of tunnleing voltage V. The conductivity is linearly proportinal to the DOS at energy eV. Plot the density of states you obtained above, and compare it to Fig. 1.

(c) Use the FD distribution to find an expression for the energy stored in the electronic states assuming $T \ll \Delta$. You can leave the answer in integral form. Assume that as long as the energy is not $\epsilon_{\vec{k}} \ll, \gg \Delta$, the density of states remains constant. For the rest of the range express your result in terms of a function $\rho(\epsilon_{\vec{k}})$ and a minimum energy $\epsilon_{\vec{k}} = 0$.

(d) What is the electronic heat capacity of the superconductor? Find the leading behavior of the heat capacity as a function of temperature upto numerical factors. Hint: check that only the regions of the spectrum closest to the fermi-energy contribute.

3. The degeneracy pressure of a Fermi gas. Consider a 3d box of volume V with free electrons of density n. What is the pressure the electrons exert on the edges of the box at $T = 0$?

   Recall that the grand canonical potential is given by: $-pV = -T \ln \mathcal{Z}$ where $\mathcal{Z} = \prod_{\vec{k},\sigma} \mathcal{Z}_k$ is the grand canonical ensemble partition function.