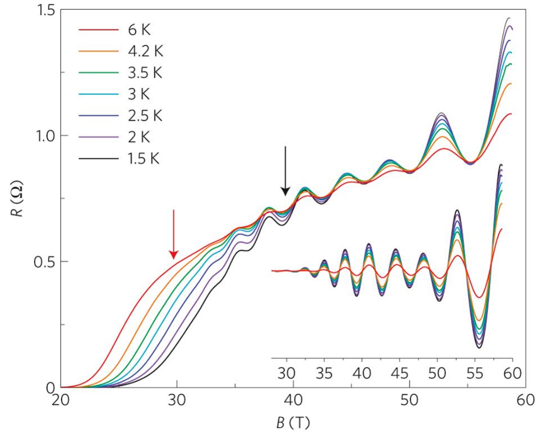

FIG. 1: A measurement by the UBC group of quantum Shubnikov de-Haas oscillations in YBCO high-T_c superconductor. These allow us to find the momentum-space size of so-called Fermi-pockets, as well as the underlying density of electrons participating in the correlated state.

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Problem set - 2

Due: Friday 18th by 5pm in TA box.

1. Field study in Shubnikov - de-Haas Quantum oscillations .

   In the paper 'Angle dependence of quantum oscillations in YBa2Cu3O6.59 shows free-spin behaviour of quasiparticles', B. J. Ramshaw, Baptiste Vignolle, James Day, Ruixing Liang, W. N. Hardy, Cyril Proust & D. A. Bonn Nature Physics volume 7, pages 234238 (2011), periodic quantum oscillations of the resistance are observed as a function of the inverse magnetic field (see Fig. 1, taken from the paper). Since YBCO is a layered material, you can assume that the measurement addressed single layer physics, and is hence a 2d measurement. It is suspected that these experiments were looking at the physics of electron pockets in the high-T_c superconductor.

   (a) What is the area in momentum-space that a pocket contains according to the measurement?

   (b) In YBCO it is thought that the oscillations arise due to 4 identical pockets. What is the (2d) electron density implied by the measurement?

   Do not try to figure out the physics of the material - this is intended as a simple exercise of the Shubnikov de-Haas effect.

2. Landau diamagnetism. As shown in class, when we apply a normal magnetic field to electrons constrained to move in a plane, the electronic states become Landau levels. Their energies are $\epsilon_n = \left(\frac{1}{2}+n\right)\hbar\omega_c$, where $\omega_c = eB/m$ is the cyclotron frequency, and B the field. Each such level has degeneracy $g = \frac{B}{h/e}$.

   The rearrangement of the Fermi disk into Landau levels increases the total energy of a non-interacting electron gas. The enery increase per area is, to lowest order in the magnetic field:

$$\Delta E/A = -\frac{1}{2}\chi_L B^2 \tag{1}$$

   χ_L (which is negative in this case) is the Landau diamagnetic susceptibility. In this problem we will calculate it.

   (a) What is the energy of the electron gas in a magnetic field B in terms of its temeperature and chemical potential? No need to evaluate the infinite sum yet. Ignore the Ze'eman spin-splitting due to the magnetic field.

(b) To evaluate the total energy, it is necessary to convert the sum to an integral. Prove the following approximation:

$$\eta \sum_{n=0}^{\infty} f(\eta(\frac{1}{2}+n)) \approx \int_{0}^{\infty} dx f(x) - \frac{\eta^2}{24} \int_{0}^{\infty} f''(x) dx \quad (2)$$

To second order in η.

(c) Using the approximation of 2b, find χ_L for a noninteracting electron gas. Assume T close to zero, but take the limit $T \to 0$ only once you obtain an expression for χ_L.