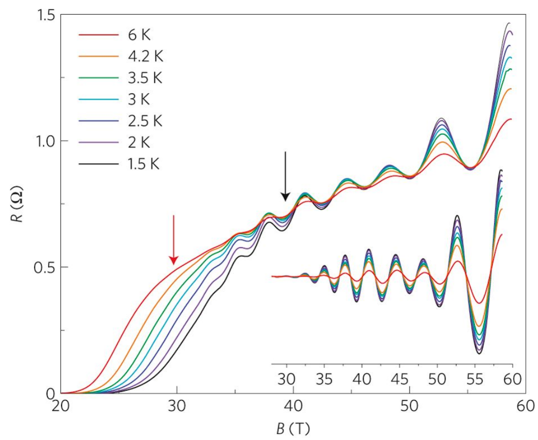

FIG. 1: A measurement by the UBC group of quantum Shubnikov de-Haas oscillations in YBCO high-T_c superconductor. These allow us to find the momentum-space size of so-called Fermi-pockets, as well as the underlying density of electrons participating in the correlated state.

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Problem set - 2

Due: Monday 22nd by 5pm in TA box.

1. Field study in Shubnikov - de-Haas Quantum oscillations .

   In the paper 'Angle dependence of quantum oscillations in YBa2Cu3O6.59 shows free-spin behaviour of quasi-particles', B. J. Ramshaw, Baptiste Vignolle, James Day, Ruixing Liang, W. N. Hardy, Cyril Proust & D. A. Bonn Nature Physics volume 7, pages 234238 (2011), periodic quantum oscillations of the resistance are observed as a function of the inverse magnetic field (see Fig. 1, taken from the paper). Since YBCO is a layered material, you can assume that the measurement addressed single layer physics, and is hence a 2d measurement. It is suspected that these experiments were looking at the physics of electron pockets in the high-T_c superconductor.

   (a) What is the area in momentum-space that a pocket contains according to the measurement?

   (b) In YBCO it is thought that the oscillations arise due to 4 identical pockets. What is the (2d) electron density implied by the measurement?

   Do not try to figure out the physics of the material - this is intended as a simple exercise of the Shubnikov de-Haas effect.

2. Boltzmann transport equation I: chemical-potential gradients.

   In class we assumed that the the electric field enters the Boltzmann equation through the $\nabla_{\vec{p}} f$ term. Alternatively, however, we could have taken into account the electric field through the chemical potential such that:

   $$\mu = \mu_0 + eEx \tag{1}$$

   Find the non-equilibrium steady state momentum-space distribution function, and the current due to this field in a free electron model with density n and relaxation time τ. Check that it is the same as what was obtained in class.

3. Boltzman transport equation II: Does magnetism make you dizzy?

   Consider a 2d metal, described by free electrons with density n and relaxation time τ. The electrons are pulled by a weak electric field $\vec{E}$, and are also subject to a magnetic field B in the direction normal to the plane of the electrons. Neglect the effect of the field on the spins (i.e., no Pauli susceptibility).

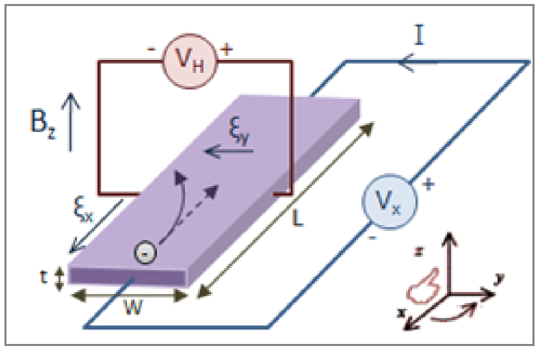

FIG. 2: A measurement of the Hall resistance R_H. Current is driven in one direction, while an electric field develops in the perpendicular direction to counter the effect of the magnetic force. Taken from Wikipedia.

(a) Write down the Boltzmann transport equation for this situation. Guidance: assume both E and B enter through the $\nabla_{\vec{p}} f$ term of the equation.

(b) What is the non-equilibrium distribution f(p,x) that arises? Hint: expect curreny in both the x and y directions. therefore $\cos\theta$ may need a partner function in superposition.

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Suppose that the 2d metal is actually confined to a strip limited in the y direction. In this case an electric field E_y will develop to make sure no current flows in the y-direction. Write and solve the Boltzmann equation for this situation.

(c) What is the emergent field E_y? Note that it is proportional to the sign of the electron charge.

(d) What is the Hall resistivity ρ_H, defined as $E_y = \rho_H j_x$? Show that it is

$$\rho_H = \frac{B}{ne}. \tag{2}$$

A measurement of the Hall resistance is shown in Fig. 2.