Ph135 - Problem Set 2

October 23, 2018

Problem 1

(a) We take the values of the magnetic field B at the minimum of the oscillations, 33.5T, 36.0T, 39.0T, 42.5T, 45.5T, 50.5T, 55.5T. We can estimate $\Delta(1/B)$ to be $0.002\mathrm{T}^{-1}$. Since

$$\Delta(\frac{1}{B}) = \frac{(2\pi)^2}{K\Phi_0} \tag{1}$$

The area in momentum space that a pocket contains is then

$$K = \frac{(2\pi)^2}{\Delta(1/B)\Phi_0} \approx \frac{39.44}{0.002 \times 2.07 \times 10^{-15}} m^{-2} \approx 9.53 \times 10^{18} m^{-2}. \tag{2}$$

(b) The electron density is given by

$$\begin{aligned} n &= (2s+1)\frac{K}{(2\pi)^2} \\ &= \frac{2K}{(2\pi)^2} \\ &\approx 4.83 \times 10^{17} m^{-2}. \end{aligned} \tag{3}$$

Problem 2

The Boltzmann transport equation in this case reads:

$$-\frac{f - f_0}{\tau} = \vec{v} \cdot \nabla f = v \cos\theta \frac{\partial f}{\partial x}. \tag{4}$$

Assume that the deviation from equilibrium is small, so we can replace f by f_0 on the RHS of the equation. We then have

$$f = f_0 - v\tau \cos\theta \frac{\partial f_0}{\partial x}. \tag{5}$$

For the steady state, we have

$$f_0 = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}, \tag{6}$$

therefore

$$\frac{\partial f_0}{\partial \mu} = -\frac{\partial f_0}{\partial \epsilon}, \tag{7}$$

and

$$\begin{aligned}\frac{\partial f_0}{\partial x} &= \frac{\partial f_0}{\partial \mu}\frac{d\mu}{dx} \\ &= -\frac{\partial f_0}{\partial \epsilon} eE,\end{aligned} \tag{8}$$

and so

$$f = f_0 + \tau v e E \frac{\partial f_0}{\partial \epsilon}\cos\theta. \tag{9}$$

The current density is given by

$$\vec{j} = 2e \int \frac{d^d p}{(2\pi\hbar)^d} \nabla_p \epsilon_p f, \tag{10}$$

so

$$\begin{aligned}\hat{x}\cdot\vec{j} &= 2\tau e^2 E \int \frac{d^d p}{(2\pi\hbar)^d}\cos\theta \frac{\partial f_0}{\partial \epsilon} \\ &= 2\tau e^2 E \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta\cos^2\theta d\theta \int \frac{k^2 dk}{(2\pi)^3}\frac{\partial \epsilon}{\partial p}\frac{\partial f}{\partial p} \\ &= e^2 \tau E \frac{k_F^2}{3\pi^2}\frac{v_F}{\hbar},\end{aligned} \tag{11}$$

which is the same as what was obtained in class.

Problem 3

(a) In this scenario, the electron experiences both the electric force and the Lorentz force, we thus have

$$(e\vec{E} + e\vec{v}\times\vec{B})\cdot\vec{\nabla}_{\vec{p}} f = -\frac{1}{\tau}(f - f_0). \tag{12}$$

(b) The Boltzmann transport equation implies

$$\begin{aligned}f &= f_0 - e\tau(\vec{E} + \vec{v}\times\vec{B})\cdot\nabla_p f \\ &= f_0 - e\tau v E \frac{\partial f_0}{\partial \epsilon}\cos\theta + e^2\tau^2 v E B \hat{\theta}\cdot\nabla_p\Big(\frac{\partial f_0}{\partial \epsilon}\cos\theta\Big) \\ &= f_0 - e\tau v E \frac{\partial f_0}{\partial \epsilon}\cos\theta + e^2\tau^2 v E B \hat{\theta}\cdot\Big(\frac{\partial}{\partial p}(\frac{\partial f_0}{\partial \epsilon}\cos\theta)\hat{p} + \frac{1}{p}\frac{\partial}{\partial \theta}(\frac{\partial f_0}{\partial \epsilon}\cos\theta)\hat{\theta}\Big) \\ &= f_0 - e\tau v E \frac{\partial f_0}{\partial \epsilon}\cos\theta - \frac{e^2\tau^2 E B}{m}\sin\theta\frac{\partial f_0}{\partial \epsilon}\end{aligned} \tag{13}$$

(c) Suppose that the 2d metal is actually confined to a strip limited in the y direction. In this case an electric field E_y will develop, which contributes a $-e\tau v E_y \sin\theta(\partial f_0/\partial\epsilon)$ term to f in (b). Therefore, to ensure that no current flows in the y direction, we must set the $\sin\theta$ term to zero, i.e.,

$$-\frac{e^2\tau^2 EB}{m}\sin\theta\frac{\partial f_0}{\partial\epsilon} = -e\tau v E_y \sin\theta\frac{\partial f_0}{\partial\epsilon}, \tag{14}$$

which implies that

$$E_y = \frac{eEB\tau}{m}. \tag{15}$$

(d) Apply the Drude result $\sigma = e^2\tau n/m$, we have

$$j_x = \sigma E = e^2 E\tau\frac{n}{m}, \tag{16}$$

so

$$\rho_H = \frac{E_y}{j_x} = \frac{eEB\tau/m}{e^2 E\tau n/m} = \frac{B}{ne}. \tag{17}$$