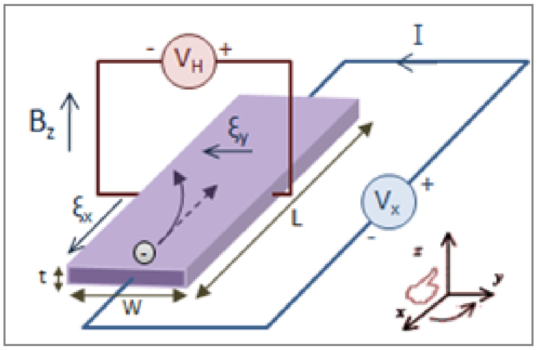

FIG. 1: A measurement of the Hall resistance R_H. Current is driven in one direction, while an electric field develops in the perpendicular direction to counter the effect of the magnetic force. Taken from Wikipedia.

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Problem set - 3

Due: Friday Nov. 1st by 5pm in TA box.

1. Boltzman transport equation: Does magnetism make you dizzy?

   Consider a 2d metal, described by free electrons with density n and relaxation time τ. The electrons are pulled by a weak electric field $\vec{E}$, and are also subject to a magnetic field B in the direction normal to the plane of the electrons. Neglect the effect of the field on the spins (i.e., no Pauli susceptibility).

   (a) Write down the Boltzmann transport equation for this situation. Guidance: assume both E and B enter through the $\nabla_{\vec{p}} f$ term of the equation.

   (b) What is the non-equilibrium distribution f(p,x) that arises? Hint: expect curreny in both the x and y directions. therefore $\cos\theta$ may need a partner function in superposition.

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   Suppose that the 2d metal is actually confined to a strip limited in the y direction. In this case an electric field E_y will develop to make sure no current flows in the y-direction. Write and solve the Boltzmann equation for this situation.

   (c) What is the emergent field E_y? Note that it is proportional to the sign of the electron charge.

   (d) What is the Hall resistivity ρ_H, defined as $E_y = \rho_H j_x$? Show that it is

   $$\rho_H = \frac{B}{ne}. \quad (1)$$

   A measurement of the Hall resistance is shown in Fig. 1.

2. Scattering-time estimates. Let's get a feeling for the times and lengths involved in electron transport in metals. Look up electrical resistivity on wikipedia. You'll find a table with the resistivity (which is the inverse of the conductivity) of various metals.

   (a) What is the most conducting metal listed, nad what is the resistivity? Likewise, what is the least conducting metal listed, and what is its resistivity?

   (b) From the resistivity data, crudely estimate the scattering time τ in the metals you listed based on our jellium model.

   (c) What is the corresponding mean free path ℓ of the metals you listed? Again, base your answer on the crude jellium model.

3. The Ioffe-Regel limit. Good metals are easy to recognize. But what about bad metals? According to the Ioffe-Regel rule, a bad metal is one where the parameter $k_F \ell \geq 1$, where k_F is the Fermi wave number, and ℓ is the mean free path. In this case, scattering is comparable to the Fermi wavelength, so thinking of the electron as a quantum particle is a stretch.

   (a) For a metal with a lattice constant of $a = 3nm$ and one electron in the conduction band per site, what would be the resistivity threshold of a 'bad-metal'?

   (b) What would be the Ioffe-Regel limit in two dimensions? Note that resistivity in 2d is given in units of resistance, which is also referred to as the resistance per square.