Ph135 - Problem Set 3

PROBLEM 1

(a) In this scenario, the electron experiences both the electric force and the Lorentz force, and thus the steady state solution in relaxation-time approach should satisfy:

$$(e\vec{E} + e\vec{v} \times \vec{B}) \cdot \vec{\nabla}_{\vec{p}} f = -\frac{1}{\tau}(f - f_0). \tag{1}$$

(b) The Boltzmann transport equation implies

$$f = f_0 - e\tau(\vec{E} + \vec{v} \times \vec{B}) \cdot \nabla_p f \tag{2}$$

Now, $|f - f_0| << f_0$, we usually approximate $\nabla_p f \sim \nabla_p f_0$. We need to be careful about this kind of approximation where the contribution from $\nabla_b f_0$ vanishes which is the case for $(\vec{v} \times \vec{B}) \cdot \nabla_p f$ term. We keep the next order contribution in this term which gives

$$\begin{aligned} f &= f_0 - e\tau v E \frac{\partial f_0}{\partial \epsilon} \cos\theta + e^2\tau^2 v E B \hat{\theta} \cdot \nabla_p \Big(\frac{\partial f_0}{\partial \epsilon} \cos\theta \Big) \\ &= f_0 - e\tau v E \frac{\partial f_0}{\partial \epsilon} \cos\theta + e^2\tau^2 v E B \hat{\theta} \cdot \Big(\frac{\partial}{\partial p} (\frac{\partial f_0}{\partial \epsilon} \cos\theta) \hat{p} + \frac{1}{p} \frac{\partial}{\partial \theta} (\frac{\partial f_0}{\partial \epsilon} \cos\theta) \hat{\theta} \Big) \\ &= f_0 - e\tau v E \frac{\partial f_0}{\partial \epsilon} \cos\theta - \frac{e^2\tau^2 E B}{m} \sin\theta \frac{\partial f_0}{\partial \epsilon} \end{aligned} \tag{3}$$

(c) Suppose that the 2d metal is actually confined to a strip limited in the y direction. In this case an electric field E_y will develop, which contributes a $-e\tau v E_y \sin\theta (\partial f_0 / \partial \epsilon)$ term to f in (b). Therefore, to ensure that no current flows in the y direction, we must set the $\sin\theta$ term to zero, i.e.,

$$-\frac{e^2\tau^2 E B}{m} \sin\theta \frac{\partial f_0}{\partial \epsilon} = -e\tau v E_y \sin\theta \frac{\partial f_0}{\partial \epsilon}, \tag{4}$$

which implies that

$$E_y = \frac{eEB\tau}{m}. \tag{5}$$

(d) Apply the Drude result $\sigma = e^2\tau n/m$, we have

$$j_x = \sigma E = e^2 E \tau \frac{n}{m}, \tag{6}$$

so

$$\rho_H = \frac{E_y}{j_x} = \frac{eEB\tau/m}{e^2 E\tau n/m} = \frac{B}{ne}. \tag{7}$$

PROBLEM 2 -ESTIMATES

(a) The most conducting metal is Silver, with resistivity $1.59 \times 10^{-8} \Omega \cdot m$. The least conducting metal is Manganese, with resistivity $1.44 \times 10^{-6} \Omega \cdot m$.

(b) We see from the Drude model that

$$\frac{ne^2\tau}{m} = \sigma, \tag{8}$$

so

$$\tau = \frac{m_e}{ne^2\rho} \approx \frac{10^{-30} kg}{10^{29} m^{-3} (1.6 \cdot 10^{-19} C)^2} \frac{1}{\rho} = 4 \cdot 10^{-22} \Omega \cdot m \cdot \frac{1}{\rho}. \tag{9}$$

For Silver, $\tau \approx 2.52 \times 10^{-14} s$; For Manganese, $\tau \approx 2.78 \times 10^{-16} s$.

(c) The mean free path for Silver and Manganese are given by:

$$\tau v_F \sim \tau \cdot 10^6 m/s \approx 10^{-8} m \text{ for Silver and } 10^{-10} m \text{ for Manganese.} \tag{10}$$

PROBLEM3 - IOFFE-REGEL LIMIT

How high could the resistance of a material be, and still be called a metal? You can't expect electrons to really behave as we described above if their mean free path is on the order of, or shorter than, their Fermi wave length. This is the Ioffe-Regel limit of bad metals. Let's estimate it.

(a) The resistivity is:

$$\rho_{IR} = \frac{m}{ne^2\tau} = \frac{mv_F}{ne^2\ell} \tag{11}$$

Now, $mv_F = \hbar k_F = h/\lambda_F$ and $4\pi/(3\lambda_F^3) = n$, and thus

$$\rho_{IR} = \frac{h}{ne^2\lambda_F^2} \sim \frac{h}{e^2}\lambda_F \tag{12}$$

It is given that the lattice constant is $3nm$ and there is one electron per site and thus $n = 1/(3nm)^3$

$$\rho_{IR} = \frac{h}{e^2}\lambda_F = \frac{h}{e^2}\left(\frac{4\pi}{3n}\right)^{1/3} \sim 10^{-5}\Omega \cdot m \tag{13}$$

(b) In 2d, we have $2\pi k_F^2/(2\pi)^2 = n \implies n\lambda_F^2 \sim 1$, and thus we would simply get a resistance:

$$\rho_{2D,IR} \sim \frac{h}{e^2} = 25k\Omega. \tag{14}$$