Ph135 - Problem Set 3

I. ESTIMATES

(a) The most conducting metal is Silver, with resistivity $1.59\times10^{-8}\Omega\cdot m$. The least conducting metal is Manganese, with resistivity $1.44\times10^{-6}\Omega\cdot m$.

(b) We see from the Drude model that

$$\frac{ne^2\tau}{m}=\sigma, \tag{1}$$

so

$$\tau=\frac{m_e}{ne^2\rho}\approx\frac{10^{-30}kg}{10^{29}m^{-3}(1.6\cdot10^{-19}C)^2}\frac{1}{\rho}=4\cdot10^{-22}\Omega\cdot m\cdot\frac{1}{\rho}. \tag{2}$$

For Silver, $\tau\approx2.52\times10^{-14}s$; For Manganese, $\tau\approx2.78\times10^{-16}s$.

(c) The mean free path for Silver and Manganese are given by:

$$\tau v_F\sim\tau\cdot10^6m/s\approx10^{-8}m\text{ for Silver and }10^{-10}m\text{ for Manganese.} \tag{3}$$

II. IOFFE-REGEL LIMIT

How high could the resistance of a material be, and still be called a metal? You can't expect electrons to really behave as we described above if their mean free path is on the order of, or shorter than, their Fermi wave length. This is the Ioffe-Regel limit of bad metals. Let's estimate it.

(a) The resistivity is:

$$\rho_{IR}=\frac{m}{ne^2\tau}=\frac{mv_F}{ne^2\ell}\to\frac{h}{ne^2\lambda_F^2}\sim\frac{h}{e^2}\lambda_F\sim10^{-5}\Omega\cdot m \tag{4}$$

(b) In 2d you would simply get a resistance:

$$\rho_{2D,IR}\sim\frac{h}{e^2}=25k\Omega. \tag{5}$$

III. THERMOELECTRIC COEFFICIENT FOR DILUTE ELECTRON GAS

In the dilute gas limit, $n\lambda_T^3\ll1$, so $\mu=T\ln(n\lambda_T^3)\ll0$, which means

$$f_0=\frac{1}{e^{(\epsilon-\mu)/T}+1}\approx\frac{1}{e^{(\epsilon-\mu)/T}}. \tag{6}$$

By the relaxation time approximation of the Boltzmann equation,

$$f=f_0-\tau\frac{\partial\vec{r}}{\partial t}\cdot\nabla_{\vec{r}}T\frac{\partial f_0}{\partial T}, \tag{7}$$

where

$$\frac{\partial f_0}{\partial T}=-\frac{\partial f_0}{\partial\epsilon}\frac{\epsilon-\mu}{T}, \tag{8}$$

and so

$$\lambda=-2e\tau\int d\epsilon\rho(\epsilon)\langle(\hat{x}\cdot\nabla_{\vec{p}}\epsilon_{\vec{p}})^2\rangle_{\text{direction}}\frac{\partial f_0}{\partial\epsilon}\frac{\epsilon-\mu}{T}. \tag{9}$$

Now

$$\rho(\epsilon) = 2 \cdot 4\pi (\frac{\sqrt{2m}}{2\pi\hbar})^3 \sqrt{\epsilon}, \tag{10}$$

and

$$\frac{\partial \epsilon}{\partial k} = \frac{\hbar^2 k}{m} = \sqrt{\frac{2\epsilon}{m}}, \tag{11}$$

we therefore have

$$\lambda = -2e\tau \int d\epsilon \frac{2^{3/2}}{3\sqrt{m}\pi^2\hbar^3} \epsilon^{3/2} \frac{\partial f_0(\epsilon)}{\partial \epsilon} \frac{\epsilon - \mu}{T}, \tag{12}$$

and

$$\lambda = \frac{2^{5/2} e\tau}{3\sqrt{m}\pi^2\hbar^3} \int \epsilon^{3/2} \frac{1}{T} \frac{1}{e^{(\epsilon-\mu)/T}} \frac{\epsilon - \mu}{T} d\epsilon = \frac{e\tau}{\sqrt{2m}\pi^{3/2}\hbar^3} e^{\mu/T} (5T^{3/2} - 2\mu T^{1/2}). \tag{13}$$

We substitute $\mu = T \ln(n\lambda_T^3)$ into the above equation, we get that

$$\lambda = \frac{10ne\tau}{m} - \frac{4ne\tau}{m} \ln(n\lambda_T^3), \tag{14}$$

where

$$\lambda_T = \left(\frac{2\pi\hbar^2}{mT} \right)^{1/2}. \tag{15}$$

Now we estimate the numerical value of the thermoelectric coefficient for a reasonably doped semiconductor, e.g., Silicon. First, let's do a dimensional analysis for λ, since λ relates current density with temperature gradient $\vec{j} = -\lambda \nabla T$, we have that $[\lambda] = (C \cdot s^{-1} \cdot m^{-2})/(K \cdot m^{-1}) = C \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$. Next, we plug in the numerical values for Silicon, $n \approx 1.08 \times 10^{18} m^{-3}$ at 300K. To estimate the scattering time τ, we use the Drude formula eq.(2), for the mass m, we need to use the effective mass of the electron in Silicon, which is $m_e^* \approx 0.2 m_e$, which is the mass that the electrons seem to have when responding to forces in the Silicon band structure. Then

$$\tau = \frac{m_e^*}{ne^2\rho} \approx \frac{2.0 \times 10^{-31}\text{kg}}{1.08 \times 10^{18}\text{m}^{-3} \times (1.6 \times 10^{-19}\text{C})^2 \times 0.1\Omega \cdot \text{m}} \approx 7.2 \times 10^{-11} s. \tag{16}$$

Hence

$$\lambda \approx \frac{10 k_B n e\tau}{m} \approx \frac{10 \times 1.38 \times 10^{-23}\text{m}^2 \cdot \text{kg} \cdot \text{s}^{-2} \cdot \text{K}^{-1} \times 1.08 \times 10^{18}\text{m}^{-3} \times 1.6 \times 10^{-19}\text{C} \times 7.2 \times 10^{-11}\text{s}}{2.0 \times 10^{-31}\text{kg}}, \tag{17}$$

and $\lambda \approx 8.6 \times 10^{-3} \text{C} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-1}$.