PH 135 final exam

Instructions:

- This exam must be taken in one sitting over the period of 5 hours from start to end. Please note on the top of your solution the starting and finishing time.
- The exam contains four problems. To obtain a perfect score you need to answer only three out of the four problems. If you tackled more than 3 problems, indicate in the beginning of your exam which problems we should grade.
- you may use any personal notes, course hand-outs, and *two* books of your choice during the exam. Note the textbooks you are using in your solution. A computer math program is not necessary, but if you feel the need to use one (mathematica, matlab, maple, etc.), it is allowed.
- If you need to make any assumptions due to lack of information in the text, specify clearly your assumptions.
- Your solution is due in Gil's mailbox by monday, Dec. 17th, 10am.
- Do not discuss the exam with your colleagues before your solution and theirs is handed in.

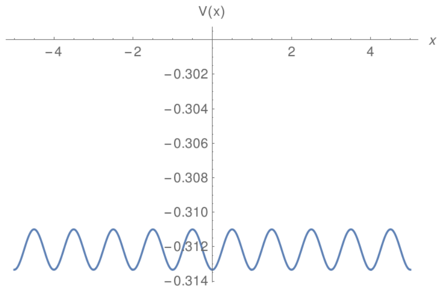

FIG. 1: The Lorentzian sum potential.

1. Multi-Lorentzian potential. Consider free electrons moving in a 1d system with kinetic energy $\hat{\mathcal{H}} = \frac{p^2}{2m}$ and subject to a weak 1d periodic potential, which is given by:

$$V(x) = -\sum_{n=-\infty}^{\infty} \frac{V}{(x-na)^2+\lambda^2}. \tag{1}$$

(see Fig. 1).

   (a) What is the periodicity of the potential?

   (b) Write $V(x)$ as an harmonic expansion:

$$V(x) = \sum_m V_m e^{imkx}. \tag{2}$$

   What are the Fourier components V_m? (what is the appropriate k?)

   (c) What is the condition on m, V, λ and a to make the potential weak?

   (d) What are the first 3 band gaps that emerge, and at what momenta do they appear?

   Useful trick:

$$\int_0^1 dx \sum_{n=-\infty}^{\infty} f(x+n) = \int_{-\infty}^{\infty} dx f(x)$$

   .

2. Berry split. Consider the double slit experiment described in Fig. 2. Electrons are emitted from point A which is equidistant to two elongated parallel slits, a distance d apart. The electrons initially have kinetic energy $\epsilon = p^2/2m$ with p and m given. Throughout, the electrons are subject to a magnetic field which affects the orientation of their spin, but whose orbital effects can be neglected. Also, we are only interested in the electrons' motion in the 2d plane of the diagram).

   The magnetic field in the two slits has a different dependence on y (with y along the slit direction):

$$B_{\pm} = -B_0 \left[\hat{z} - r\left(\hat{x}\cos(2\pi y/L) \pm \hat{y}\sin(2\pi y/L)\right)\right]. \tag{3}$$

   With the $\pm$ indicating the left (-) and right (+) slits. The field outside the slits is $B = B_0(\hat{z} + r\hat{x})$, and assume that the electrons in this problem are initally pointing along the $\hat{z} + r\hat{x}$ direction and that $r \ll 1$.

   The electrons finally interfere on a screen (B) at some large distance (compared to the separation between the slits).

   The change of the magnetic field in the two slits is going to produce a Berry phase that will shift the interferene pattern of the electrons emerging from the two slits. Assume that the field is sufficiently large that the time evolution of the electronic spins is adiabatic.

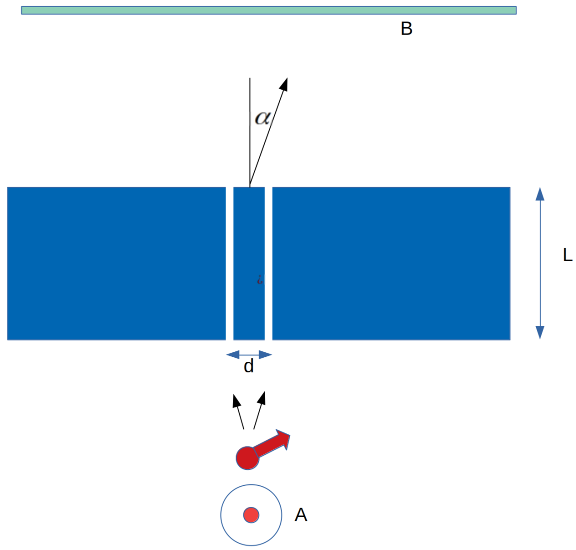

FIG. 2: Electrons emitted from point A interfere on screen B by going through two elongated slits. The Zeeman magnetic field in the slits varies as a function of distance along the slits, which we mark as the y direction.

(a) Warmup. If there was no Zeeman field present At which angle α in the plane of the diagram would there be constructive interference?

(b) Berry effects. What is the Berry phase that each of the paths incurs due to the winding magnetic field? At which α will the maxima occur when the Berry phase is taken into account?

(c) What is the condition on B_0, r, p, m such that the adiabatic assumption indeed applies?

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Throughtout you can rely on equations derived in class.

3. Cold and neutral graphene. Graphene could be considered as having two copies of the following Hamiltonian:

$$\hat{\mathcal{H}} = v(p_x\sigma^x + p_y\sigma^y) \tag{4}$$

Every momentum has two energy eigenvalues (in each copy). At charge neutrality the chemical potential is $\mu = 0$. In this problem you are asked to find the conductivity of graphene as a function of temperature T in the relaxation time approximation.

(a) What are the energy eigenvalues $\pm\epsilon_{\vec{p}}$ for each momentum $\vec{p}$?

(b) Write down the Boltzmann equation for electrons subject to an electric field $\vec{E} = E\hat{x}$. Assume a relaxation time τ which is independent of momentum magnitude or direction, and of temperature. It may be convenient, although not essential, to write down the occupation distribution separately for positive and negative energy electrons: $f_+(\vec{p})$, $f_-(\vec{p})$. Also, assume the form of the driven distribution function as:

$$f_\pm(\vec{p}) = f_\pm^{(0)}(p) + \delta f_\pm(\vec{p}) \tag{5}$$

with $f_\pm^{(0)}$ marking the undriven distribution function, which is the Fermi function.

(c) What is $\delta f_\pm(\vec{p})$ to first order in τ?

(d) What is the total current carried by the electrons subject to the electric field? You can leave your answer here in integral form.

(e) Show that the temperature dependence of the conductivity is of the form $\sigma \propto T^\gamma$. What is γ?

4. Chern band motion. Consider an electron in the lowest band of a Chern insulator. The energy dispersion of the electron is:

$$\epsilon_{\vec{p}} = E_0 - m(\cos p_x + \cos p_y) \tag{6}$$

with $-\pi \le p_{x,y} < \pi$ giving the Brillouin zone of the band. Also, the band possesses a uniform band curvature $\vec{\Omega} = \frac{1}{2\pi}\hat{z}$.

(a) Write the semiclassical equations of motion for the electron. Assume that it is subject to an electric field $\vec{E} = E\hat{x}$ which is too weak to excite the electron to higher bands. Don't forget the anomalous velocity term! (No need to derive the equations, and don't be concerned about the sign of the anomalous velocity).

(b) Solve the equations of motion to find the localtion of the electron as a function of time, assuming it is initially at $\vec{p}(t = 0) = 0$.