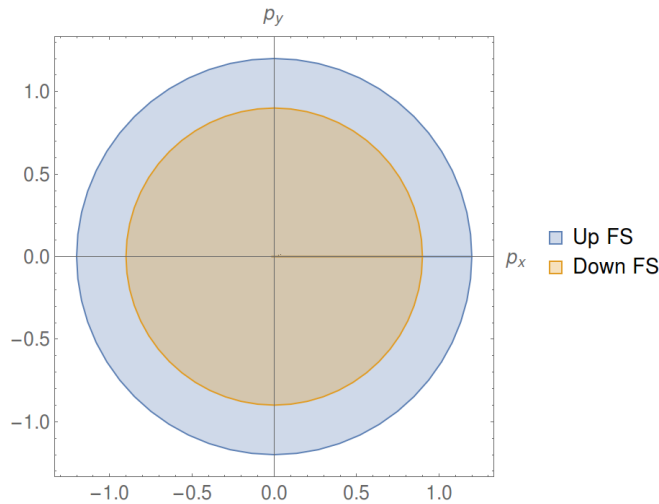
FIG. 1. A magnetic field splits the Fermi surfaces of the two spins, and there are more spins pointing along the field than against it.The electrons on the outer annulus give rise to a net spin polarization and magnetization.

II. PHYSICAL PROPERTIES OF A DEGENERATE ELECTRON GAS

Last lecture we derived the Fermi-Dirac distribution and just barely started calculating with it. Today we will derive the main physical properties of an electron gas at low temperature. We will calculate the zero-temeprature magnetic susceptibility, and also the heat capacity at low temperatures. The thing that we will see over and over again, is that the properties of the electron gas are completely determined by what's going on near the fermi surface. Everything else is frozen out. This makes the physics of electrons quite intuitive - please keep your eyes peeled, and see how this occurs.

A. Pauli Susceptibility of an electron gas

Another important property of the zero-temperature electron gasses is the Pauli-spin-Susceptibility. When we turn on a magnetic field, the spin Hamiltonian gets out of deep freeze, and we need to put a term for it in the hamiltonian:

$$\hat{\mathcal{H}}_{spin} = -g\mu_B \vec{H} \cdot \hat{\vec{\sigma}} = -g\mu_B H \sigma_z \tag{9}$$

choosing the spin-quantization access along the magnetic field. Now, spin up has less energy then spin down. The chemical potential of the electron gas is still the same for the two spins, but this implies that the fermi energy - which here we distinguish from the chemical potential by saying that it is the *kinetic* energy upto which electron states are filled - is not the same for the two electron flavors.

For down electrons, the total energy is:

$$E_\downarrow = \epsilon_{\vec{k}} + Hg\mu_B \tag{10}$$

while

$$E_\uparrow = \epsilon_{\vec{k}} - Hg\mu_B \tag{11}$$

Since the energy shift for up is minus that of down, some electrons from the high-energy down spins, would flip to point in the direction of the magnetic field. This will happen until we have

$$E_{F\uparrow} - Hg\mu_B = E_{F\downarrow} + Hg\mu_B \tag{12}$$

So, as you can see from Fig. **??**, we have an excess of upspins, and we can easily calculate how many:

$$n_\uparrow - n_\downarrow = \int\limits_{\mu - Hg\mu_B}^{\mu + Hg\mu_B} \rho(E)dE = 2\rho(E_F)Hg\mu_B \tag{13}$$

Where $\rho(E)$ here is the single-spin DOS. The excess spin magnetization is then:

$$g\mu_B(n_\uparrow - n_\downarrow) = 2\rho(E_F)H(g\mu_B)^2 \tag{14}$$

So the susceptibility is:

$$\chi_{Pauli} = 2\rho(E_F)(g\mu_B)^2 \tag{15}$$

It is a direct measure of the density of states at the Fermi surface. Free electron spins tend to point in the direction of the field, but don't actually have magnetization in zero field. Hence this is called **paramagnetism**.

Now, in addition to this effect there is also going to be an orbital effect on the general magnetization, and the question rises - how do we measure the Pauli susceptibility alone? This is done using the Knight shift - The energy level of nuclear spins are coupled to the electronic spins through the contact term of the hyper-fine hamiltonian of an atom. This coupling is much stronger than the coupling of nuclei to a raw magnetic field (since their Bohr magneton is three orders of magnitude smaller than that of the electron's). Using NMR one can get very accurate readings of nuclear energy levels, and there one see's precisely this quantity.

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[1] E. M. Purcell, R. V. Pound, Phys. Rev. **81**, 279 (1951).