ph 135

Problem set - 6

Due: Nov. 25th by 5pm in TA box.

1. Dresselhaus vs. Rashba. Consider a 2d quantum well where electrons obey the following kinetic Hamiltonian:

$$H = \frac{p^2}{2m} \tag{1}$$

In addition, they are subject to Dresselhaus interaction:

$$H_D = \beta(\sigma^x p_x - \sigma^y p_y) \tag{2}$$

and Rashba interaction:

$$H_R = \alpha(\sigma^x p_y - \sigma^y p_x) \tag{3}$$

   (a) What are the energy eigenvalues for electrons with momentum $\vec{p}$?

   (b) Show that the electronic energy states split into two non-degenerate (never crossing) bands for all α and β except for when $\alpha = \beta$, and $\vec{p} = 0$.

   (c) What is the winding of the spin (the total number of turns the electron spin makes as we probe it about the $p = 0$ point, with counterclockwise defined as positive) as a function of α and β? To answer it is useful to think of extreme cases.

2. (a) A spin 1 undergoes a rotation from the $m = 1$ state to the $m = -1$ state and back in the following slow process:

$$|\psi\rangle = \frac{1}{2}\begin{pmatrix} 1+\cos(t) \\ \sqrt{2}\sin(t) \\ 1-\cos(t) \end{pmatrix} \tag{4}$$

   with $t = 0$ at first, and $t = 2\pi$ in the end. What is the Berry phase accumulated in the process?

   (b) What if instead, the wave function underwent a similar procedure with the following function,which involves a rotation about the z-axis:

$$|\psi\rangle = \frac{1}{2}\begin{pmatrix} (1+\cos(t))e^{it} \\ \sqrt{2}\sin(t) \\ (1-\cos(t))e^{-it} \end{pmatrix} \tag{5}$$

   and only go through a quarter of the cycle, with $t = 0$ at first, and $t = \pi/2$ at the end?