ph 135

Problem set - 7

Due: Dec. 9th by 5pm in TA box.

1. Distorted Jackiw-Rebbi mode.

   Consider the effective model for the Thouless pump with the potential shift parameter ϕ:

   $$\hat{\mathcal{H}} = \sigma_z v\hat{p} + g\cos\phi(x)\sigma^x + g\sin\phi(x)\sigma^y. \tag{1}$$

   The system at hand has a domain wall between two shift paramters:

   $$\phi(x) = \begin{cases} -\gamma/2 & x < 0 \\ \gamma/2 & x > 0 \end{cases} \tag{2}$$

   Find the localized mode at the domain wall. What is its energy and wave function? Hint: Assume that the space dependence is a simple exponential on both sides.

2. Landauer conductance formula. One of the intermediate steps in the derivation of the Thouless pump considered the effective model for an electronic single-band:

   $$H = \sigma^z v\hat{p}. \tag{3}$$

   Now, suppose that the chemical potential for left movers and right movers are difference by the energy $e \cdot V$. How much net current is flowing through the wire? Some instructions: (a) Recall that the current is the velocity times the density, and that the total density is given by an integral over momentum with the right coefficients. (b)You only need to account for states that are uncompensated.

   Show that the conductance (ratio of current to voltage) is velocity-independent, and equals:

   $$\sigma = e^2/h. \tag{4}$$

   This is the Landauer conductance per balistic (no scattering) channel. Compute the resistance corresponding to $\sigma = e^2/h$.