ph 135

Problem set - 7

Due: Dec. 3rd by 5pm in TA box.

1. Carriers and chemical potential in n-type semiconductor.

   Consider an n-type semiconductor with donor denisty N_D, and the donor states appearing an energy δ below the bottom of the conduction band. Assume that the density of states $\rho(\epsilon) = \rho_0$ is constant for the conduction band, and that the valence band is so far in energy that it could be neglected.

   (a) Show that the chemical potential obeys the following equation:

   $$\rho_0 T \ln\left(1 + e^{\mu\beta}\right) = N_D \frac{1}{e^{(\delta+\mu)\beta} + 1} \tag{1}$$

   with μ measured from the botom of the conduction band.

   (b) Find an approximate expression for μ when the temperature is small - $T \ll \delta$.

   (c) Find an approximate expression for μ when the temperature is large - $T \gg \delta$.

   (d) Provide a qualitative plot of the chemical potential and the number of carrier as a function of T. If you'd like to solve the problem numerically, reasonable parameters are: $\rho_0 \sim 10^{28} \frac{1}{m^3 eV}$ and $N_D \sim 10^{23} \frac{1}{m^3}$, and $\delta \sim 0.05eV$.

   (e) What is the number of carriers as a function of T for the two limits?
   Hint: $e^{\mu\beta} \ll 1$ in all circumstances.

2. Dresselhaus vs. Rashba. Consider a 2d quantum well where electrons obey the following kinetic Hamiltonian:

   $$H = \frac{p^2}{2m} \tag{2}$$

   In addition, they are subject to Dresselhaus interaction:

   $$H_D = \beta(\sigma^x p_x - \sigma^y p_y) \tag{3}$$

   and Rashba interaction:

   $$H_R = \alpha(\sigma^x p_y - \sigma^y p_x) \tag{4}$$

   (a) What are the energy eigenvalues for electrons with momentum $\vec{p}$?

   (b) Show that the electronic energy states split into two non-degenerate (never crossing) bands for all α and β except for when $\alpha = \beta$.

   (c) What is the winding of the spin (the total number of turns the electron spin makes as we probe it about the $p = 0$ point, with counterclockwise defined as positive) as a function of α and β? To answer it is useful to think of extreme cases.